VOLUME 1, ISSUE 10

APPLIED AND INDUSTRIAL MATHEMATICS M.S.

APIM NEWS LETTER SPRING 2023

EDITOR: EMMY SMITH

WHAT'S INSIDE

PROGRAM SUMMARY

This program is designed to deliver advanced knowledge and skills in the field of applied mathematics, preparing you for careers in a variety of rapidly growing fields, including data science, data analysis, statistics, business or government intelligence, market research, operations research, and consulting.

In this program, you will receive a comprehensive education in applied mathematics, comprising a blend of theoretical and practical knowledge, with emphases on differential equations, mathematical modeling, machine learning, numerical computations, optimization, and statistical modeling and data analytics.

You'll select one of two tracks focused on a high-demand area in business and industry:

- differential equations/optimization track
- applied statistics/mathematical finance track

FALL 2023 COURSES

Registration begins: April 10th, 2022

600-LEVEL

DIFFERENTIAL EQUATIONS/ OPTIMIZATION CONCENTRATION

MATH 635: APPLIED NUMERICAL ANALYSIS

3 CREDIT HOURS

Direct and iterative methods for solving linear systems. Newton's method. Solution of ordinary differential equations, including Euler's method, Runge-Kutta methods, Adams-Bashforth methods and Adams-Moulton methods. Stability analysis. Finite difference methods for the solution of partial differential equations. **Prerequisites**: MATH 374 or MATH 574, and MATH 435 or MATH 535, or consent of chairperson.

These are the 600 level courses you can find all the 500-level courses in the TU catalog.

APPLIED STATISTICS/ MATHEMATICAL FINANCE CONCENTRATION

MATH 646: STATISTICAL THEORY II/ REGRESSION ANALYSIS

3 CREDIT HOURS

Theoretical and applied aspects of regression analysis including linear regression, generalized linear models, model selection, multicollinearity, leverage points, transformations, AIC, BIC, AICC, ANOVA tests, serially correlated errors, logistic regression, deviance, and simple models for stationary time series.

Prerequisites: MATH 330 or equivalent.

MATH 639: LOSS MODELS

4 CREDIT HOURS

Severity models, frequency models, aggregate models, survival models, construction of parametric models, and credibility models.

Prerequisites: MATH 532 or equivalent.

ALUMNI INTERVIEW

INTERVIEW WITH

Mark Evans | Mathematics Teacher at the Science and Mathematics Academy

WHAT MADE YOU DECIDE TO PURSUE A MASTER'S IN APPLIED MATHEMATICS?

Early in my career as a public-school teacher, I began working on the planning of a new STEM oriented magnet program for Harford County Public Schools, which became the Science and Mathematics Academy (SMA). With my teaching focus shifting to advanced students at the SMA, the benefits of a degree in my content area led me to pursue a Master’s in Applied and Industrial Mathematics. I wanted to challenge myself with learning more advanced content within my field and use that deeper knowledge in the classroom.

WHAT WOULD YOU RECOMMEND TO STUDENTS LOOKING TO FIND AN AREA FOR RESEARCH?

Students looking to find an area of research will face many challenges. Therefore, they should choose something that they are passionate about.

HOW DID THE SKILLS FROM TU CLASSES HELP PAVE YOUR WAY INTO YOUR CAREER?

Although, I already had a career before pursuing my master’s degree, the challenging coursework gave me the skills to tackle further independent learning and write curricula for the SMA, which is not taught in other high schools. The research project I completed has now become the cryptology course that I teach. This course involves teaching motivated high school students the mathematics behind ciphers and cryptanalysis.
Students looking to find an area of research will face many challenges. Therefore, they should choose something that they are passionate about.

WHAT STOOD OUT TO YOU ABOUT TU?

TU interested me because it was a local university that had a flexible enough schedule for a full-time teacher to complete a degree that was not in education.

WHAT ADVICE WOULD YOU PROVIDE CURRENT TU APIM STUDENTS?

To ensure timely completion of your degree and to meet all necessary requirements, it is important to stay organized and work closely with your advisor, particularly if you are currently employed full-time.

A GEOMETRIC INEQUALITY ARISING FROM A MULTIDIMENSIONAL OPTIMAL RECOVERY PROBLEM

DAWN MYERS
DR. SERGIY BORODACHOV | FACULTY ADVISOR

Errors in the approximation of a function is an ever expanding area of growth in mathematics. Machine learning, artificial intelligence and data science are just a few areas where intense research is being conducted and there is a continual need to develop new methods to reduce the error and dene upper bounds for various classes of functions. In the eld of approximation theory, the smoothness of a function or a class of functions can dene the eciency or optimality of the approximation.

Bernstein-Bézier (BB) polynomials have become a centerpiece of research in spline interpolation as they are useful tools for constructing piecewise polynomial and parametric surfaces defined over triangulated planar domains, [4]. Since the 1960s and the advent of computer generated design, BB polynomials have played an extremely important role in CAGD (computer-aided geometric design), data tting and interpolation, computer vision, and elsewhere, [1]. In the formation of spline interpolation, BB polynomials are used as the building blocks and correspond to domain points.

A number of problems in approximation theory that ask for an optimal algorithm of recovery of functions on a multidimensional domain (such as a triangle, a simplex, or more generally, a convex polytope) based on discrete data (values and partial derivatives at the vertices), reduce to proving a certain type of geometric inequalities. These inequalities involve powered distances from a point in the domain to the vertices of the domain, the barycentric coordinates of the point (or some generalization of them), and the circumradius of the domain. The power to which the distances are raised represents the smoothness of the class in the corresponding optimal recovery problem.

The power to which the distances are raised is represented by in our inequality below. Our research focused on finding a numerical approximation for the maximum, , in the inequality. In the equation, we take the sum of the weighted distances raised to a certain power and determine whether the value is less than or equal to the circumradius of a d-dimensional simplex raised to the same power. We dene

$$\sum_{i=0}^{d} b_i |v - v_i|^{\alpha} \le R^{\alpha},\ v \in T \qquad (1)$$

where T is a regular d-dimensional simplex with vertices $v_0, \dots, v_d$ and $b_0, \dots, b_d$ are the barycentric coordinates of point v relative to T and R is the circumradius of T.

Our numerical research was based on a well-known geometric inequality for $\alpha = 2$, [5]. In 2018, Dr. Sergiy Borodachov published his findings on the worst-case error for the class of three times differentiable functions on a convex polytope inscribed in a sphere of radius R, expanding this inequality to the power 3, ($\alpha = 3$), [3], [2]. Borodachov's method of approximation used a quasi-interpolating recovery method he and T. S. Sorokina developed in 2011. Borodachov and Sorokina [4] constructed a spline method which is optimal for recovery of multivariate classes of twice differentiable functions f defined on a bounded convex polytope, using information (values and gradients of f) at a set of nodes. This "...optimal method of recovery was given by a quadratic interpolating spline over a Delaunay triangulaion of X in R^d", [5]. Borodachov's 2018 research expanded the use of the optimal recovery algorithm on a class of functions on a simplex of smoothness r = 3, (dierentiability of the class), from data sampled at the vertices of smoothness, less by one, [2], [4].

In our numerical research, we estimated the conjectured highest value of for $2 \le d \le 9$. This inequality (1) is known to hold on any simplex of any dimension with positive power 3 or less and on a regular simplex with power 4 or less in dimension 3 or higher. It fails with power 6 (and even 5.94) on any simplex in any dimension, proved in [2]. Our computations show that the lowest dimension in which the inequality may hold on a regular simplex with the $\alpha = 5$ is dimension 9.

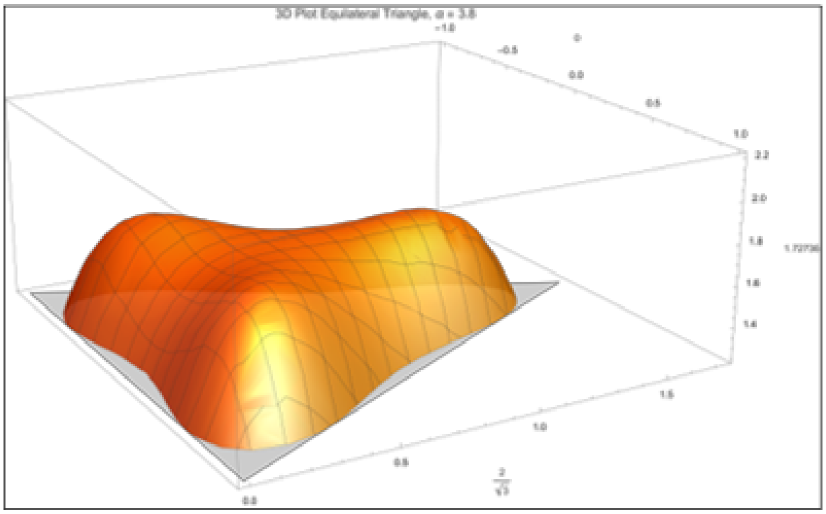

Figure 1. 3D image of simplex with d = 2 (triangle). α = 3:8 shows 3 peaks between the centroid and each vertex and the critical point at the centroid. The peaks between the centroid and the vertices are all symmetric. This plot was made using Mathematica.

For the computational project, we dene the function representing the left hand side of the inequality (1) using regular simplices with sides equal to 2. In dimension 2, our simplex is an equilateral triangle. In dimension $3 \le d \le 9$, we restrict the left-hand side of the inequality to a line segment joining the center and vertex. We then produce a plot of the univariate function. Using a 3D Mathematica plot, for d = 2, we provide evidence that the function achieves its maximum on the line joining the center with the vertices.

Rigorous analytic proofs for the upper bound of are available in several cases. In our research, we conjectured that for every dimension d, the largest exponent in the inequality (1) is achieved on a regular simplex. In the plane we estimated its value numerically. For 3 d 9, the value we found is an upper bound. We conjecture the maximum of the left-hand side is achieved on a regular simplex on line segments joining the circumcenter and the vertices.

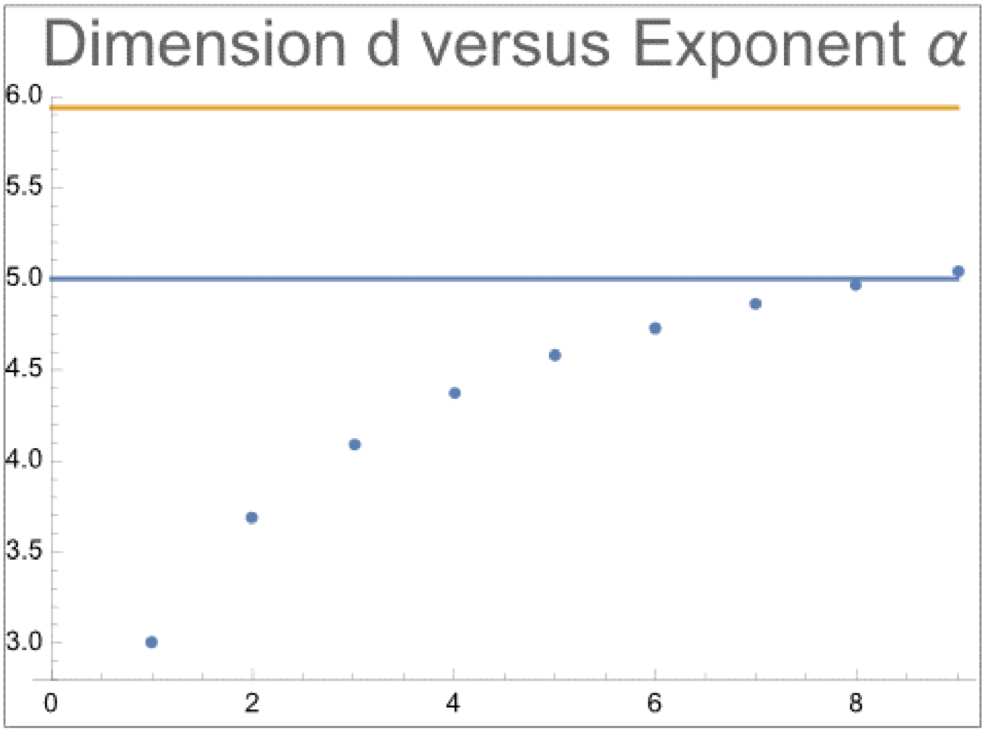

Figure 2. This graph shows the plot of the upper bound for maximal for dimension 2 through 9, with the value for d = 2 being a rounded estimate. Notice the plotted sequence is concave downward and increasing. In the graph we nd numerical evidence that the sequence of maximal 's as a function of dimension is increasing, concave downward, $\alpha > 4$ for $d \geq 3$ and $\alpha > 5$ for $d \geq 9$; provided the conjecture is true. There is a horizontal asymptote at $\alpha < 5{:}94$. This is the upper bound. Our inequality (1) doesn't hold on any simplex in any dimension when = 5:94.

References

[1] Alfeld, Peter, Neamtu, Marian, & Schumaker, Larry L. 1996. Bernstein-Bézier polynomials on spheres and sphere-like surfaces. Computer Aided Geometric Design, 13(4), 333--349.

[2] Borodachov, S V. 2022. Inequality for the convex combination of powered distances to the vertices of a convex polytope. June.

[3] Borodachov, Sergiy. 2018. Optimal recovery of three times dierentiable functions on a convex polytope inscribed in a sphere. Journal of Approximation Theory, 234, 51--63.

[4] Borodachov, Sergiy, & Sorokina, Tatyana. 2011. An optimal multivariate spline method of recovery of twice dierentiable functions. BIT Numerical Mathematics, 51(3), 497--511.

[5] Liu, Jian. 2012. Some new inequalities for an interior point of a triangle. Journal of Mathematical Inequalities, 195--204.

ADMISSION REQUIREMENTS

- A baccalaureate degree in mathematics or a related field from a regionally accredited college or university. The applicant's undergraduate training must have included at least three terms of calculus, a course in differential equations, and a course in linear algebra. Students with any deficiency in their mathematical background may be admitted conditionally if they are willing to correct such deficiency.

- An undergraduate GPA of at least 3.00 for full admission and at least 2.75 for conditional admission is required. All GPA calculations for admission are based upon the last 60 credits of undergraduate and post-baccalaureate study.

SPRING 2023	SUMMER 2023	FALL 2023
Priority Deadline: **Nov. 1**	Final Deadline: **May 1**	Priority Deadline: **June 15**
Final Deadline: **Jan. 7**		Final Deadline: **Aug. 15**

Master's Degree

Applied and Industrial Mathematics

FOR MORE INFORMATION

Please Contact

Dr. Yunwei Cui

Department of Mathematics

7800 York Road, Room 231

Towson, MD 21252

410-704-3245

410-704-4149

ycui@towson.edu

t.ly/yw86

TOWSON UNIVERSITY | Mathematics