Optimizing a rocket bell nozzle diverging region for three combustion gas gammas

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Introduction

Since the early 20th century, rocketry has been the sole method of reaching orbital flight and extraterrestrial entities. Building rockets with these capabilities is a substantial engineering optimization problem of high cost, risk, and difficulty. Numerous subsystems are optimized to achieve efficient spaceflight, but none as important as propulsion.

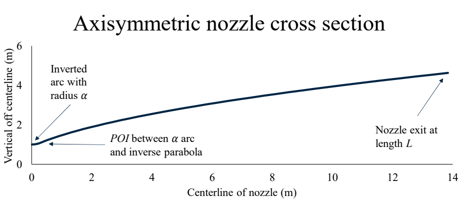
The propulsion subsystem consists of fuel and oxidizer combusted in a chamber, and then accelerated through a nozzle. The nozzles geometry is carefully shaped such that it abides by the governing fluid dynamics equations, the compressible Navier Stokes equations, to maximize the momentum imparted to the exhaust gases. The change in momentum of the exhaust gases from chamber to exit, is equal to the impulse imparted to the rocket.

Optimization of the nozzle geometry for one gas gamma (ratio of specific heat at constant pressure to that at constant volume) can be performed iteratively through Rao's method (Rao, 1958) or by the method of characteristics applied to the governing partial differential equations (Khare & Sahe, 2021). These methods are restrained to a single gas gamma and ideal gas conditions (inviscid and adiabatic).

This study optimized a diverging arc-parabolic nozzle geometry across 1.2, 1.4, and 1.6 combustion gas gamma values to create a flex-fuel efficient nozzle. To account for multiple gas gammas, the methodology utilized computational fluid dynamics (CFD) to gain simulated thrust performance results for each combustion gas gamma. Then, with model development and optimization, an optimized flex-fuel nozzle was pinpointed.

Materials and Methods

The diverging arc-parabolic nozzle geometry was defined by three geometric optimization parameters (Graph 1): initial arc radius (α), centerline point of inflection (*POI*), and the nozzle length (*L*). Across all nozzles and situations, the throat radius and exit radius were held constant. Based on the above constants and parameters a set of constraint equations were solved to determine the defining functions of the nozzle geometry. The weight is determined through a centerline revolution with a constant nozzle thickness and density.



Graph 1 (left): Diverging arc-parabolic nozzle geometry of 0.3 α, 0.031 m *POI*, and 1.53 m *L*. Arrows indicate variables place of effect. The throat radius is 0.1156 m and exit radius 0.535 m.

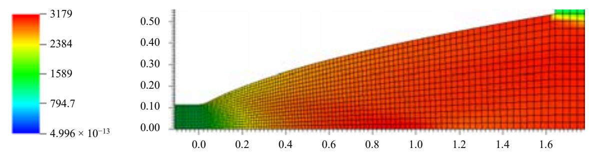
Materials and Methods (continued)

The CFD software utilized to simulate ideal, compressible, and supersonic fluid flow was OF (OpenFOAM). OF utilizes the numerical finite volume method to solve the governing partial differential equations to predict fluid flow through time and 3D space.

The specific solver chosen for this research was rhoCentralFoam: a transient, compressible, and turbulent solver. A hexahedral mesh was required by the numerical solver and generated by OF (Figure 1). This open-source code was run with an SMA computing hours allocation from the National Science Foundation on the Expanse supercomputer at the San Diego Supercomputer Center. An external code was implemented to yield thrust results on the nozzle.

For each gas gamma, a separate set of OF configuration files were setup with corresponding ideal gas properties and inlet conditions. Inlet conditions of pressure, Mach, and temperature were consistent across the gammas. A C program was written that calculates discrete spline points of a nozzle based on the three design optimization parameters (α, *POI*, *L*). The code also formats the mesh configuration file with these spline points and vertices of the mesh regions.

Figure 1 (below): Centerline vertical cross section of 3D hexahedral wedge mesh. Mesh graph axis units in m. Pseudo color overlay of exhaust gas speed with scale in m/s to the left. Example of flow output from OF and visualization in Visit.

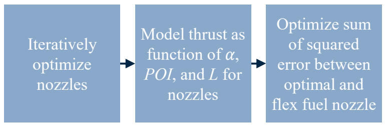


For each gamma, a rough optimal nozzle was found by optimizing a performance variable (Thrust – 2 × Weight) against its geometry. Then a 4D space of inputs: α, *POI*, and *L*, and outputs: performance variable, was filled surrounding each optimum with 27 data points for all gammas. A 4D quadratic model was fitted to each gamma (Expression 1). This model was optimized to find an optimal geometry for each gamma situation. Figure 2 shows the full optimization process.

$$\sum_{i=0}^{2}\sum_{j=0}^{2}\sum_{k=0}^{2} c_{ijk}\alpha^i POI^j L^k$$

Expression 1 (left): 27 coefficient model with degree two and three independent variables α, *POI*, and *L*. The model predicts the nozzles performance variable.

Figure 2 (right): Optimization process. Last step requires minimizing squared performance error = $(\text{Optimal}_{1.2} - Model_{1.2})^2 + (\text{Optimal}_{1.4} - Model_{1.4})^2 + (\text{Optimal}_{1.6} - Model_{1.6})^2$ to obtain an optimal flex-fuel nozzle.



Results

The quadratic models overpredicted the optimal performance variable in each gamma situation when compared to the true simulated performance given by OF (Table 1).

Optimal nozzle performance			
	1.6	1.4	1.2
α	0.660	0.667	0.509
POI (m)	0.0355	0.0360	0.0327
L (m)	1.713	1.643	1.48
Model (N)	193,923	209,076	244,909
OpenFOAM (N)	193,012	207,466	237,955
Percent error	−0.472%	−0.776%	−2.92%

Table 1 (left): Results of optimizing quadratic model for performance variable against α, *POI*, and *L* for each gas gamma. Simulated OF performance of each optimal geometry also displayed. Error between model prediction and OF performance results.

The optimal flex-fuel nozzle was found to have a geometry of α = 0.524, *POI* = 0.0319 m, and *L* = 1.600 m through minimization of the function in Figure 2. The flex-fuel nozzles performance in OF was comparable to that predicted by the model (Table 2).

Model flex-fuel nozzle performance			
	1.6	1.4	1.2
Model (N)	192,909	207,603	244,027
Percent difference flex-fuel to optimal	−0.524%	−0.707%	−0.361%
OpenFOAM flex-fuel nozzle performance			
OpenFOAM (N)	192,900	207,384	237,539
Percent difference flex-fuel to optimal	−0.0580%	−0.0392%	−0.175%

Table 2 (left): Flex-fuel nozzle performance calculated by the quadratic model and OF. Flex-fuel nozzle performance compared to the optimal nozzle performance for each gamma. Flex-fuel degradation in performance is greater for the model prediction compared to OF calculation.

Conclusions

The research was able to generate an optimal flex fuel nozzle geometry with less than 1% performance degradation compared to the optimal nozzle geometry for 1.2, 1.4, and 1.6 gammas, meaning no significant performance is lost while gaining the benefits of reusability, adaptability, and inter-planetary travel. Future research can focus on optimizing fuel choice without significant concern for nozzle geometry.

It must be noted the performance results generated are for the diverging region of the nozzle, not the converging region. Further research could optimize a converging region for the flex fuel nozzle and merge it with the diverging region developed here.

The 27 points chosen for the model were too coarse, causing slight inaccuracies in the model predictions compared to OF.

References

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Rao, G. V. R. (1958). Exhaust nozzle contour for optimum thrust. *Journal of Jet Propulsion*, *28*(6), 377–382. https://doi.org/10.2514/8.7324